Lecture 06
EN4562 Autonomous Systems Introduction to Fuzzy Logic Control

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Acknowledge: De Montfront University, UK

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Fuzzyness in Real World

- "As complexity increases, precise statements lose meaning and meaningful statements lose precision."

  - Lofti Zadeh UCBerkley

- "So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality."

  - Albert Einstein

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Contents

Problem Statement

- Many complex plants/processes are manually controlled by experienced operators
- Transferring knowledge of a complex plant/process to a control algorithm is difficult

Logical Solution

- Model operator's control action instead of modeling the plant/process
- Implement control process as rules, not as differential equations

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What is Fuzzy Logic

- A way of getting computers to make decisions more like a human would.
- It uses **fuzzy sets** and **fuzzy rules** to model the world and to make decisions about it.
- Fuzzy sets allows us to handle situations that are not precise.
  - Hot weather ⇒ at what temperature the decision is made?
  - Big house ⇒ at what floor area the decision is made?
  - Tall man ⇒ at what height the decision is made?

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Grocery Store Keeper

- How much potatoes should be ordered?
- Too little ⇒ run out of stock ⇒ loose sales
- Too much ⇒ throw away the excess ⇒ loose profit

- He has to consider lot of different uncertain factors (season, whether, holiday ahead….)

Such situations arise very often in real world, where a definite decision has to be made despite situation uncertainty

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Fuzzy Sets

- Fuzzy logic is one of the ways such decisions can be made using computers
- To make fuzzy decisions, **fuzzy sets** and **fuzzy rules** are needed
- A **set** is a collection of related items. A **crisp set** is collection of items that belong to that set completely, against **fuzzy set**, which is a collection of items which belong to that set to **different degrees**.

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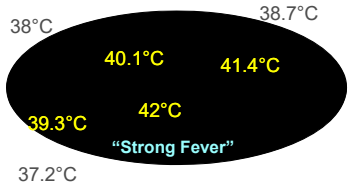
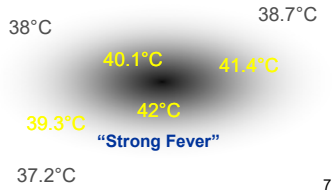
Crisp and Fuzzy Sets

- Crisp Set



- Fuzzy Set



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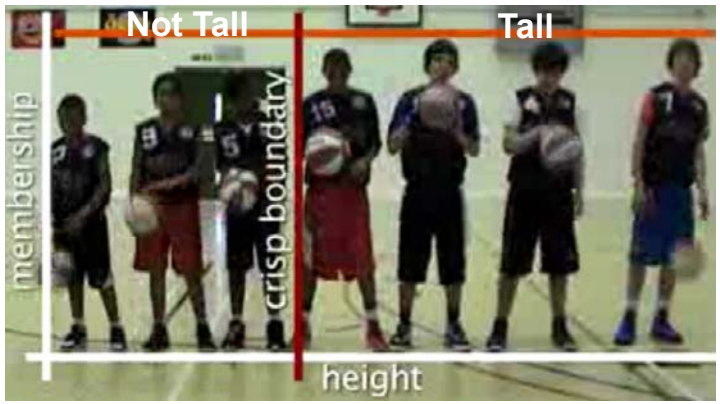
Crisp Set "Tall"

- Anyone who is over a certain (crisp) height is tall
- Result is either **tall** or **not tall**



- Not always the best way to classify information

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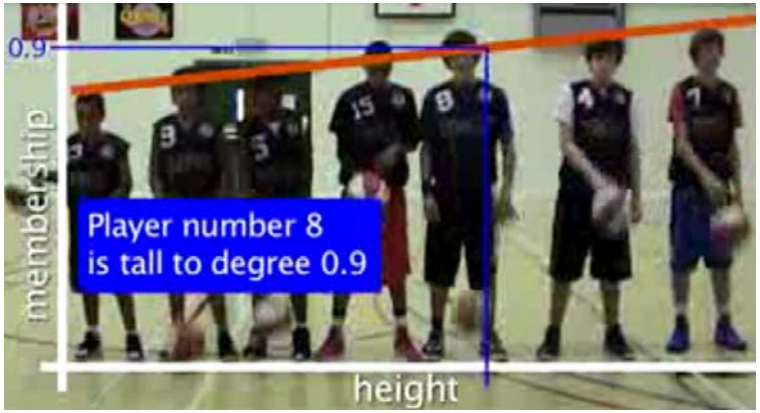
Fuzzy Set "Tall"

- Assume everyone is tall to some degree [0,1]



- Using fuzzy logic in categorizing items gives more details about the items in the set

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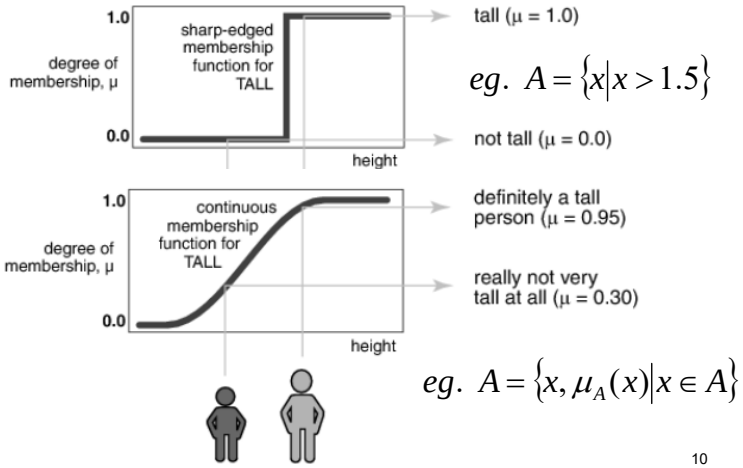
Crisp and Fuzzy Set "Tall"



$$eg.\ A = \{x \mid x > 1.5\}$$

$$eg.\ A = \{x, \mu_A(x) \mid x \in A\}$$

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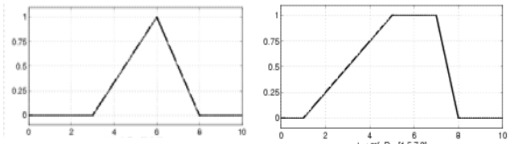
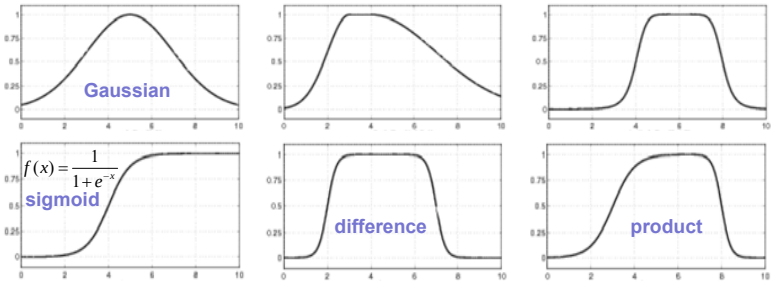
Membership Function Shapes

- Piecewise linear : Triangular and Trapezoidal

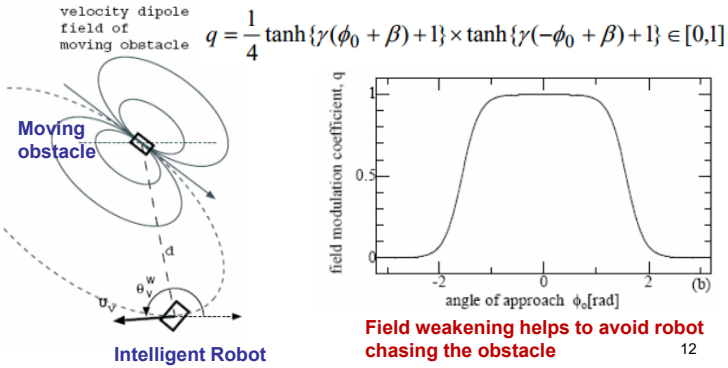


- Nonlinear: Gaussian, and Sigmoid (logistic fn, hyperbolic tangent fn)



Membership Function Shapes

- Hyperbolic tangent functions $\tanh(x) = \frac{1-e^{-2x}}{1+e^{-2x}}$
- Velocity dipole field method: Avoidance of moving obstacles



Field weakening helps to avoid robot chasing the obstacle

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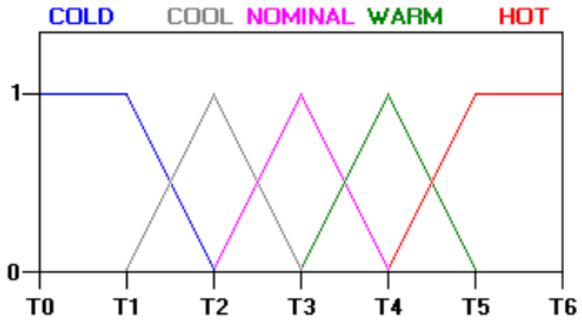
Fuzzy Sets (Membership Functions)

- For temperature



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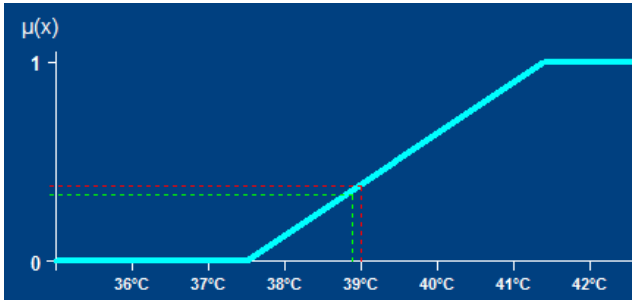
Membership to a Fuzzy Set

- Membership Values:

  $\mu_X(35°C) = 0$ $\mu_X(38°C) = 0.10$ $\mu_X(41°C) = 0.9$
  $\mu_X(36°C) = 0$ $\mu_X(39°C) = 0.35$ $\mu_X(42°C) = 1.0$
  $\mu_X(37°C) = 0$ $\mu_X(40°C) = 0.65$ $\mu_X(43°C) = 1.0$



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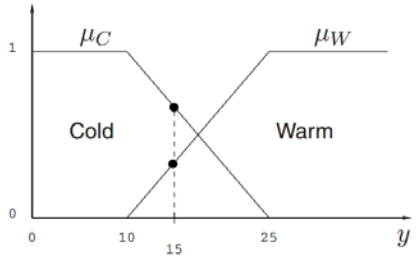
Fuzzification (Multiple Simultaneous Memberships)

$y = 15$: $\mu_C(15) = 2/3$ and $\mu_W(15) = 1/3$



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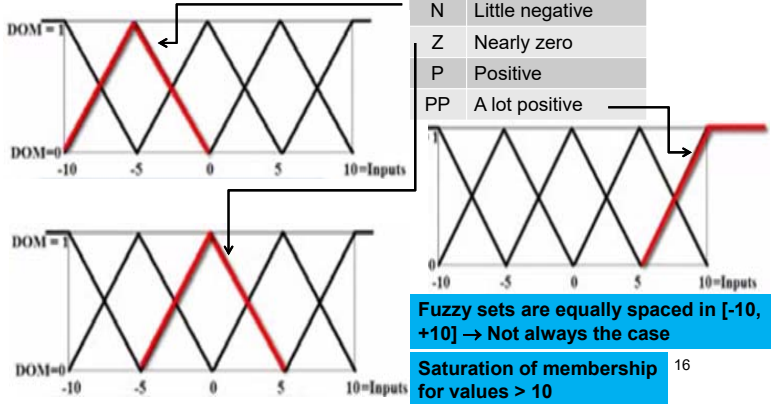
Fuzzification (input fuzzy sets)

- Converting a crisp value into a fuzzy value (degree of membership) using five fuzzy input sets
- Input range [-10, 10] eg. error

NN	A lot negative
N	Little negative
Z	Nearly zero
P	Positive
PP	A lot positive



Fuzzy sets are equally spaced in [-10, +10] → Not always the case

Saturation of membership for values > 10

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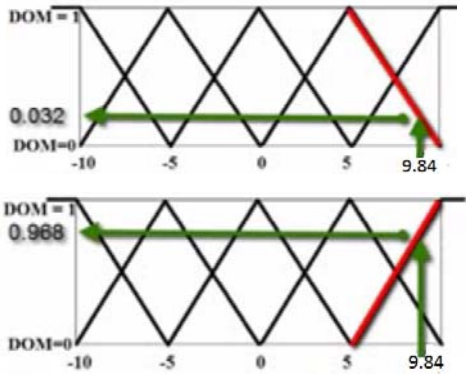
Fuzzification (input fuzzy sets)

- Eg: Input value = 9.84 using linear interpolation on each fuzzy set results DoM as follows



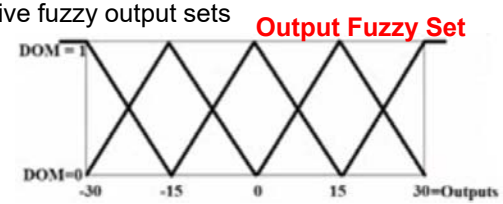
Fuzzy Set	DoM
NN	0
N	0
Z	0
P	0.032
PP	0.968

Fuzzy Rules

- **Fuzzy rules combine partially true (fuzzy) facts**
  - **IF** a person is **"tall" AND "agile" THEN he should** consider **"basketball" as a sport.**
  - **IF** a person is **"short" AND "broad" THEN he should** consider **"wrestling" as a sport.**
- **How fuzzy rules work**
  - Take a person and test to what degree he is tall and to what degree he is agile and then decide to what degree he should consider basketball as a sport
  - "Tall", "Agile", "Short", "Broad" are Input fuzzy sets, whereas "Basketball", "Wrestling" are output fuzzy sets

Defuzzification: output fuzzy sets

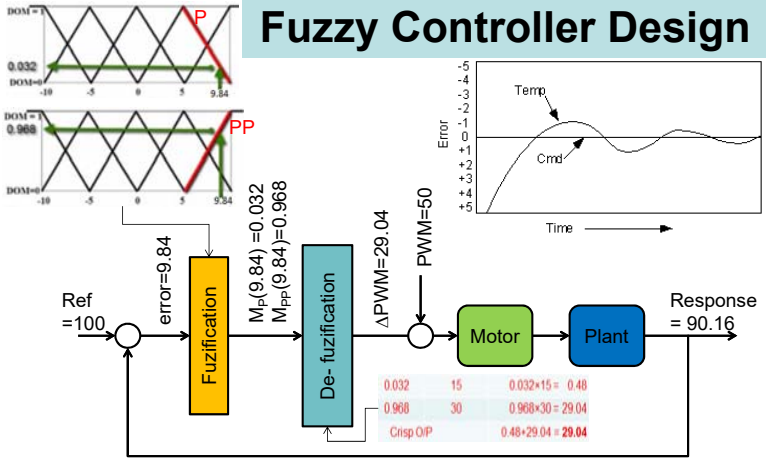
- Converting a fuzzy value (degrees of membership) into a crisp value using five fuzzy output sets
- Output range is [-30, 30]
  eg: motor shaft position



Set	I/P DoM	O/P set Val	O/P Component
NN	0	-30	0×-30 = 0
N	0	-15	0×-15 = 0
Z	0	0	0×0 = 0
P	0.032	15	0.032×15 = 0.48
PP	0.968	30	0.968×30 = 29.04
Crisp O/P			0.48+29.04 = **29.04**

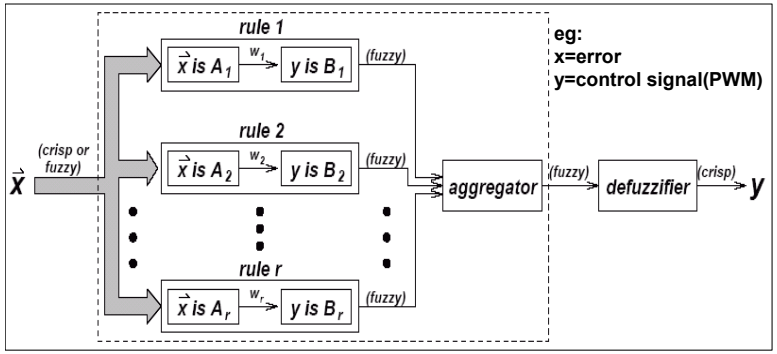
Output set value is the peak or the knee height of a fuzzy set

Fuzzy Controller Design

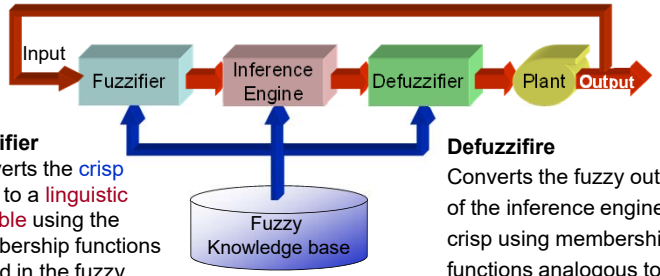


- Feedback control system would work as desired once fuzzy sets (membership functions) are properly designed
- It is hard to analyze system's behavior, stability etc..

Fuzzy System Architecture



Fuzzy Control System



Fuzzifier
Converts the crisp input to a linguistic variable using the membership functions stored in the fuzzy knowledge base.

Inference Engine
Using If-Then type fuzzy rules converts the fuzzy input to the fuzzy output.

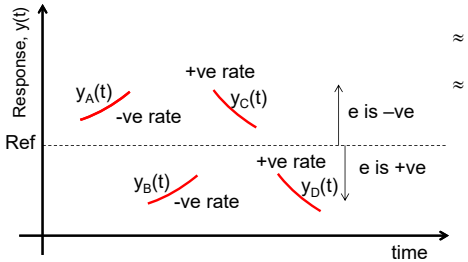
Defuzzifire
Converts the fuzzy output of the inference engine to crisp using membership functions analogous to the ones used by the fuzzifier.

Zadeh, L.A., "Outline of a new approach to the analysis of complex systems and decision processes," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 3, No. 1, pp. 28-44, Jan. 1973.

Control Rules

$$e(k) \approx \text{Ref} - y(k)$$

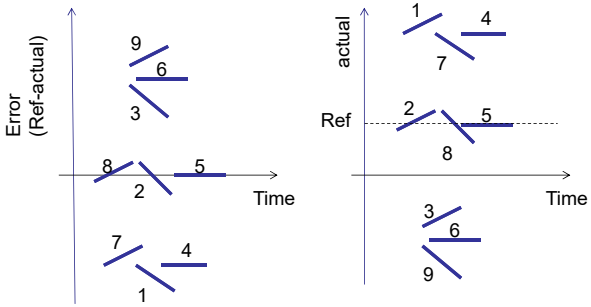
$$\dot{e}(k) \approx \frac{e(k)-e(k-1)}{T} \approx \frac{[\text{Ref}-y(k)]-[\text{Ref}-y(k-1)]}{T} \approx \frac{y(k-1)-y(k)]}{T}$$



State Identification ⟹ Control Action

- State A: e is –ve AND de/dt is –ve → State A: -ve
- State B: e is +ve AND de/dt is -ve → State B: No control?
- State C: e is –ve AND de/dt is +ve → State C: No control?
- State D: e is +ve AND de/dt is +ve → State D: +ve

Determination of Control Action



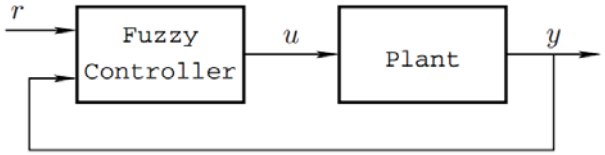
1	C
2	H or NC?
3	H
4	C
5	NC
6	H
7	C
8	C or NC?
9	H

1. IF Cmd-Temp=N AND d(Cmd-Temp)/dt=N THEN Output=C
2. IF Cmd-Temp=Z AND d(Cmd-Temp)/dt=N THEN Output=H
3. IF Cmd-Temp=P AND d(Cmd-Temp)/dt=N THEN Output=H
4. IF Cmd-Temp=N AND d(Cmd-Temp)/dt=Z THEN Output=C
5. IF Cmd-Temp=Z AND d(Cmd-Temp)/dt=Z THEN Output=NC
6. IF Cmd-Temp=P AND d(Cmd-Temp)/dt=Z THEN Output=H
7. IF Cmd-Temp=N AND d(Cmd-Temp)/dt=P THEN Output=C
8. IF Cmd-Temp=Z AND d(Cmd-Temp)/dt=P THEN Output=C
9. IF Cmd-Temp=P AND d(Cmd-Temp)/dt=P THEN Output=H

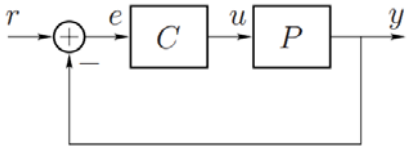
Error rate \ error	N	Z	P
N	1 C	2 H	3 H
Z	4 C	5 NC	6 H
P	7 C	8 C	9 H

Fuzzy and Conventional Control

- Fuzzy Controller: Model the control action of an expert operator

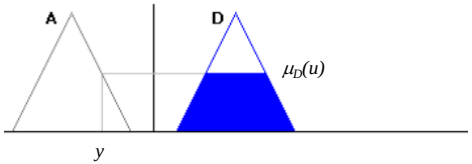


- Conventional Controller: Model Plant and then synthesize the controller

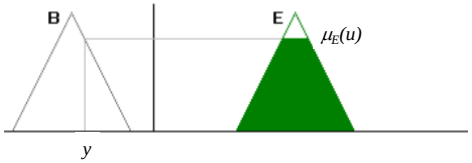


Mamdani Fuzzy Inference

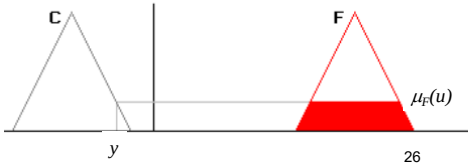
Rule 1: If *y* is A THEN *u* is D

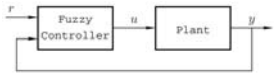


Rule 2: If *y* is B THEN *u* is E



Rule 3: If *y* is C THEN *u* is F

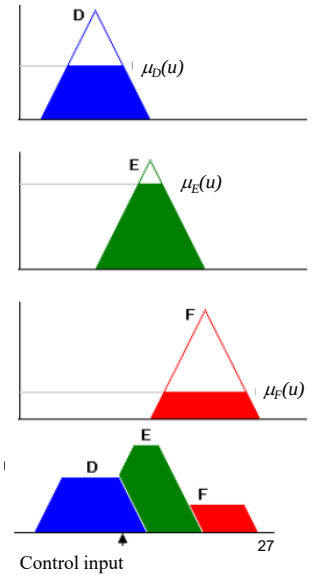




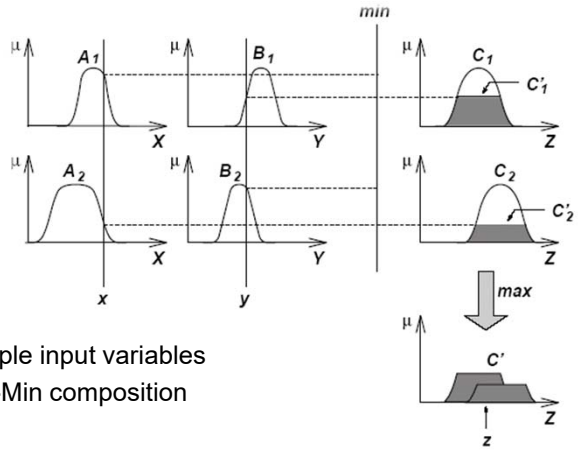
D, *E*, and *F* are fuzzy sets of control input *u* *A*, *B*, and *C* are fuzzy sets for output *y*

Defuzzification

- Aggregate Rule outputs using centroid method



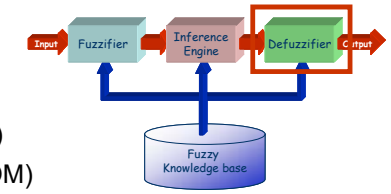
Combining Multiple Fuzzy Variables



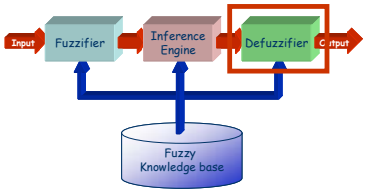
- Multiple input variables
- Max-Min composition

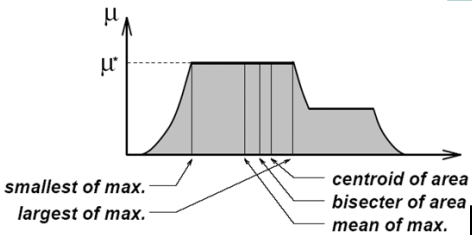
Defuzzifier

- Five commonly used defuzzifying methods:
  - Centroid of area (COA)
  - Bisector of area (BOA)
  - Mean of maximum (MOM)
  - Smallest of maximum (SOM)
  - Largest of maximum (LOM)



Defuzzifier





$$z_{COA} = \frac{\int_Z \mu_A(z) z dz}{\int_Z \mu_A(z) dz},$$

$$\int_{\alpha}^{z_{BOA}} \mu_A(z) dz = \int_{z_{BOA}}^{\beta} \mu_A(z) dz,$$

$$z_{MOM} = \frac{\int_{Z'} z dz}{\int_{Z'} dz},$$

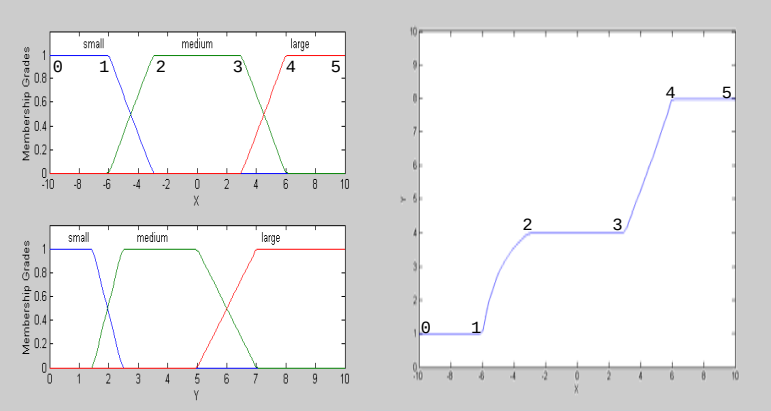
$$where\ Z' = \{z; \mu_A(z) = \mu^*\}$$

Example

X = input $\in [-10, 10]$
Y = output $\in [0, 10]$

R1 : If X is small then Y is small
R2 : If X is medium then Y is medium
R3 : If X is large then Y is large

Max-min composition and centroid defuzzification were used.



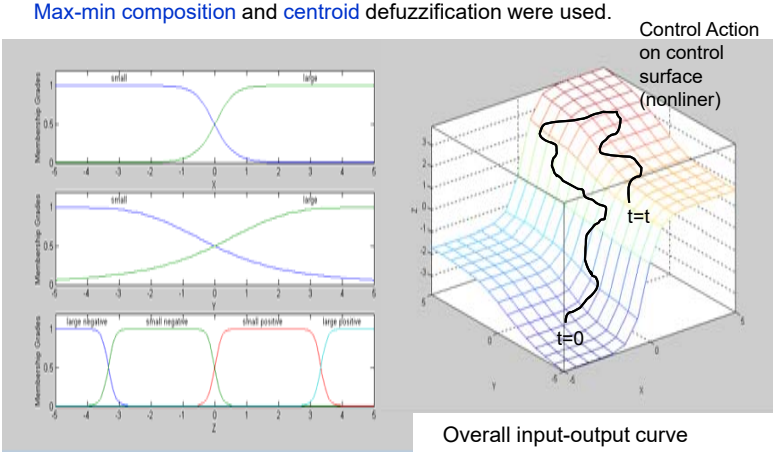
Overall input-output curve

Example

$X, Y, Z \in [-5, 5]$

R1: If X is small & Y is small then Z is negative large
R2: If X is small & Y is large then Z is negative small
R3: If X is large & Y is small then Z is positive small
R4: If X is large & Y is large then Z is positive large

Max-min composition and centroid defuzzification were used.



Overall input-output curve

Sugeno Fuzzy Models

- Also known as TSK fuzzy model
  - Takagi, Sugeno & Kang, 1985
- Goal: Generation of fuzzy rules from a given input-output data set.

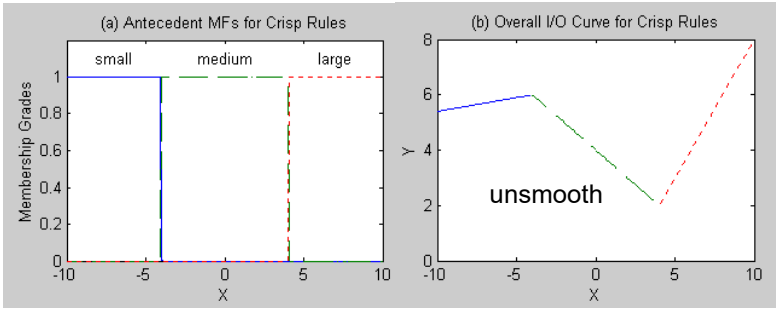
If X is A and Y is B then $z = f(x, y)$

Input Fuzzy Sets Output Crisp Function

$f(x, y)$ is very often a polynomial function of x and y.

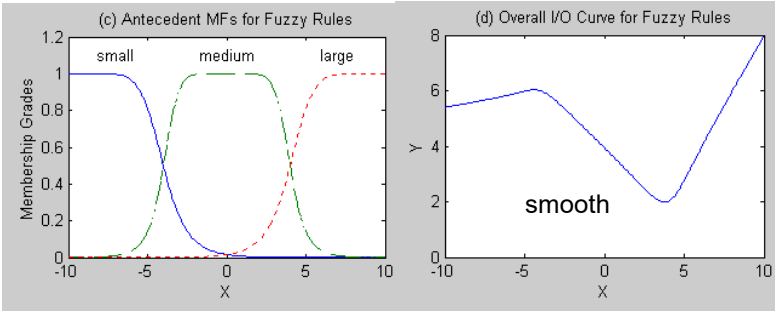
Example: Crisp Set

R1: If X is small then $Y = 0.1X + 6.4$
R2: If X is medium then $Y = -0.5\,X + 4$ X = input $\in [-10, 10]$
R3: If X is large then $Y = X - 2$



With Fuzzy Sets

R1: If x is small then $Y = 0.1X + 6.4$
R2: If x is medium then $Y = -0.5X + 4$ x = input $\in [-10, 10]$
R3: If x is large then $Y = X - 2$



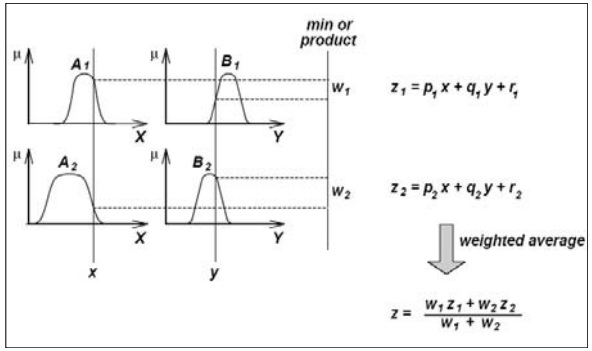
Fuzzy sets make the overall input-output curve a smoother one.

Example (TSK Fuzzy Control)

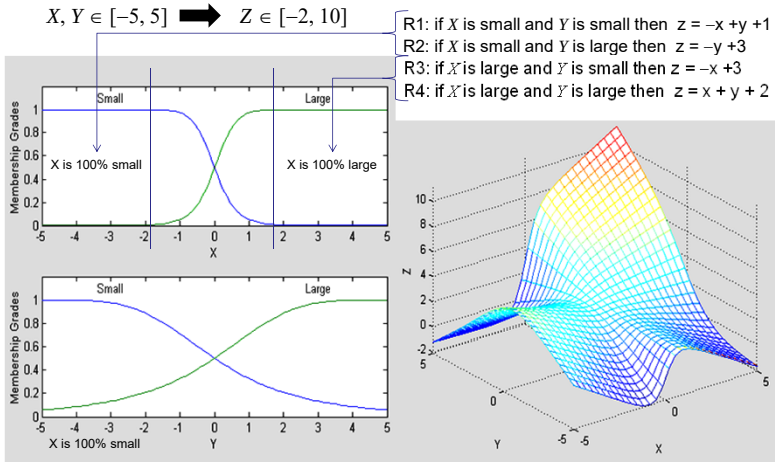
Rules

R1: if X is small and Y is small then $z = -x + y + 1$
R2: if X is small and Y is large then $z = -y + 3$
R3: if X is large and Y is small then $z = -x + 3$
R4: if X is large and Y is large then $z = x + y + 2$

Inference Engine

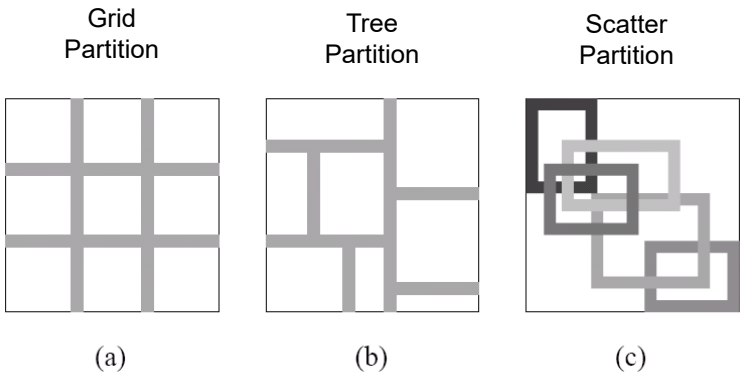


TSK Two Variable Fuzzy Control



input-output mapping plane

Partition Styles for Input Space



Is Fuzzy Control Really Fuzzy?

- The word “fiuzzy” in Oxford Dictionary is defined as “blurred”, “indistint”, “imprecisely defined”, “confused”, and “vague”.
- However, fuzzy systems are to be precisely defined. The fuzzy set geometry, fuzzification, fuzzy inferencing, and defizzification, are not imprecise or vague at all.
- Fuzzy control is a special kind of nonlinear control method, which too is to be defined precisely
- In fuzzy control, the word “fuzzy” is used only as a technical adjective, analogous to the word “linear” used in linear control theory (constant coefficients in the sysem’s differential equation)

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Why Fuzzy? (Approximate Reality)

- The real world, its systems and processes are too complicated to be modeled precisely.
- Therefore, fuzziness (approximation of reality) must be introduced to obtain reasonable, and tractable model.
- All theories in engineering are approximations of the real world. A good engineering theory should be precise to the extent that it characterizes the key features of the system/process, and be tractable for mathematical analysis.
- Fuzzy theory precisely describe the nonlinear features of the system/process, but may not be appealing for mathematical analysis.
- Fuzzy is the area in mathematics, where human heuristics can be incorporate into modeling and control.

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Why Fuzzy? (Handling Nonlinearity)

- Nonlinearity: efficient linear systems are too restrictive (operational point), having a narrow operational range. To operate in a wider range needs nonlinear modeling and control which is computationally extensive, and suffer from stability problems (saturation, backlash).
- Plant uncertainty: System dynamics vary with time, temp, etc.
- Temporal behavior: Systems in general show a varying behavior and time delay in feedback. These variations are difficult to be modeled and to be incorporated into the controller.
- Multivariate, multi-loops cause serious modeling and control problems.

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Fuzzy Control Features

- Fuzzy controllers are robust (in view of noise and disturbances), and able to operate in a wider dynamic range of the system/process. Single FLC can replace multiple linearized controllers designed for a given plant.
- Developing a FLC is cheaper than developing a model-based controller (intuitive versus analytical)
- FLC are customizable. Rules can be added, modified, or deleted looking at the performance.
- FLC can incorporate into it expert or heuristic knowledge expressed in linguistic form (IF …Then…)
- It is easy to learn and understand a FLC and how it operates.

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FLC Applications

- **Consumer Products**
  - Washing machines, cookers, camcoders, vacuum cleaners, microwave ovens, word translators
- **Systems**
  - Elevators, cranes, trains, automotives (engine, transmissoin, brakes)
- **Software**
  - Medical diagnosis, security, data compression

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